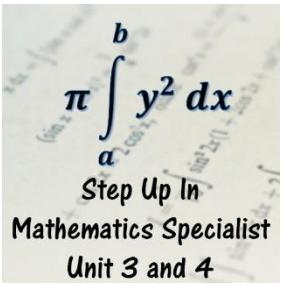
5.5 Modelling Motion

Problems Worksheet



1. Acceleration is defined as the rate of change of velocity with respect to time. I.e. $a = \frac{dv}{dt}$.

   a. Demonstrate $a = v\frac{dv}{dx}$.

   b. Demonstrate $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$.

2. Determine whether the following objects are undergoing simple harmonic motion about the origin. In each case, x is the position in metres relative to the origin, v is the velocity in metres per second, $\ddot{x}$ is the acceleration in metres per second squared and t is the time in seconds.

   a. $x = 3\cos 2t + 1$

   b. $x = 12\cos \pi t + 5\sin \pi t$

c. $x = 3\sin \pi t$

d. $v = 2x^2 - 3x$

3. An object is moving with simple harmonic motion such that its acceleration $\ddot{x}$ in metres per second squared is given by $\ddot{x} = -4x$, with x the displacement in metres. It is known that when $t = 0$ the object is at its maximum positive displacement of $3\ m$.

    a. Use integration techniques to determine an expression for the displacement of the object in terms of time t in seconds. Do not use any formulae from the data sheet to assist your derivation.

b. Calculate exactly the total distance travelled by the object between $t = 0$ and $t = \sqrt{2}\pi$ seconds.

c. Calculate exactly the displacement of the object between $t = 0$ and $t = \sqrt{2}\pi$ seconds.

d. Denote the origin O, the point of maximum positive displacement A and maximum negative displacement B. Determine exactly the time between the object being located at B and the midpoint of OA.

4. The end of a spring is moving according to simple harmonic motion with amplitude 8 cm from some origin and period of 2 s. It is initially situated with a positive displacement of $4\sqrt{2}$ cm and a positive velocity.

   a. Write two equations for the displacement of the end of the spring x in terms of time t. Write one in terms of the sine ratio and one in terms of the cosine ratio.

   b. Calculate the percentage of time the end of the spring is at least 3 cm from the origin.

5. A Ferris wheel has as wheel diameter of 150 m and is constructed so that its minimum point is 10 m above the ground. It takes 24 minutes to complete an entire lap. Calculate to the nearest ten seconds, the shortest and longest time between a cabin being 50 m above the ground.